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Gluonic penguin contribution to B $\to \pi\pi$ decays

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Abstract. We use the light-cone sum rule technique to calculate the contribution of the gluonic penguin operator O_{8g} to the decay of the B-meson to two pions. Leading-order perturbative and non-perturbative corrections are included, corresponding to hard and soft exchanged gluons, respectively. While the overall contribution of this operator to the decay is small as expected before, we find that the so-called soft-gluon part of this contribution is of the order of the hard-gluon one. This implies that the inclusion of soft gluons in the calculation of $B \to \pi\pi$ matrix elements may be important.

PACS. 13.25.Hw Hadronic decays of mesons: Decays of bottom mesons – 11.55.Hx Sum rules

1 Introduction

The hadronic decays of B-mesons are a source of information on CP violation and the parameters of the CKM mixing matrix, in particular the angles α and γ. Together with the present intensive experimental investigation of the B physics, this makes the $B \to \pi\pi$ decays interesting.

Hadronic decays of bottom mesons are not easily tractable within the framework of perturbation theory alone. One starts from an effective Hamiltonian and computes the Wilson coefficients and parametrizes the matrix elements of the relevant operators. The main difficulty lies in the accurate evaluation of the matrix elements, which are essentially non-perturbative. Recently QCD factorization has been suggested and applied [1] to the computation of these matrix elements. It is able to provide answers in the limit of infinite b-quark mass, but the question remains whether finite-mass effects affect the results significantly.

In this contribution, we present the results of a recent calculation [2] of the gluonic penguin operator matrix element contributing to the $B \to \pi\pi$ decay. Our approach to the problem relies on the QCD (light-cone) sum rules [3–6]. The framework for this calculation was introduced in [7]. This method allows the inclusion of both hard and soft gluons in a consistent manner, so that their respective contributions can be consistently compared to each other. Importantly enough, the soft gluons often evade other methods, like constituent quark models or factorization. On the other hand, our result shows that they cannot be neglected since at a finite value of the b-quark mass they are responsible for a large part of the matrix element of the gluonic penguin operator. Although the overall role of this particular operator in the description of the decay considered is not overwhelming, this fact is nevertheless an indication that similar sum rule analyses may be worthwhile for other operators. We have begun with O_{8g} due to its relative computational simplicity as compared to other terms of the effective Hamiltonian.

In the following, we present the method used to evaluate the matrix element $\langle B|O_{8g}|\pi\pi\rangle$ based on light-cone sum rules and define the appropriate correlation function. We then present the results and the various contributions appearing in the evaluation of this element, including the comparison between the hard- and soft-gluon part. Numerical predictions are also given. Finally, we present the conclusions.

2 Method

The $B \to \pi\pi$ decay is described by the effective Hamiltonian,

$$H_W = \frac{G_{\mathrm{F}}}{\sqrt{2}}\left\{\lambda_u\left[\left(c_1(\mu)+\frac{c_2(\mu)}{3}\right)O_1(\mu)+2c_2(\mu)\widetilde{O}_1(\mu)\right] + \cdots + \lambda_t c_{8g}(\mu)O_{8g}(\mu)\right\}, \quad (1)$$

where $\lambda_u = V_{ub}V_{ud}^*$, $\lambda_t = V_{tb}V_{td}^*$ and we have displayed only the most important operators and the gluonic penguin operator O_{8g}, which is of our interest in this paper:

$$O_1 = \left(\bar{d}\Gamma_\mu u\right)\left(\bar{u}\Gamma^\mu b\right), \quad \widetilde{O}_1 = \left(\bar{d}\Gamma_\mu \frac{\lambda^a}{2}u\right)\left(\bar{u}\Gamma^\mu \frac{\lambda^a}{2}b\right), \quad (2)$$

$$O_{8g} = \frac{m_b}{8\pi^2}\bar{d}\sigma^{\mu\nu}\left(1+\gamma_5\right)\frac{\lambda^a}{2}g_s G^a_{\mu\nu}b, \quad (3)$$

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where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$ and m_b is the b-quark mass. In order to find the matrix element $\langle B|O_{8g}|\pi\pi\rangle$, we use the correlation function

$$F_\alpha(p,q,k) = -\int \mathrm{d}^4x\,\mathrm{d}^4y e^{-i(p-q)x+i(p-k)y} \times\langle 0|T\left\{j^{(\pi)}_{\alpha 5}(y)O_{8g}(0)j^{(B)}_5(x)\right\}|\pi^-(q)\rangle, \quad (4)$$

where the quark currents $j^{(\pi)}_{\alpha 5} = \bar{u}\gamma_\alpha\gamma_5 d$ and $j^{(B)}_5 = m_b\bar{b}i\gamma_5 d$ interpolate π- and B-mesons, respectively. This correlation function is evaluated in the spacelike region of the variables $s_1 = (p-k)^2$, $s_2 = (p-q)^2$, and $Q^2 = (p-q-k)^2$. Then the result is analytically continued and matched with the double dispersion relation to give a sum rule for the desired matrix element, see [2, 7] for details. After Borel transformations in the variables s_1 and s_2, the final formula for the matrix element is [7]

$$\left\langle\pi^-(p)\pi^+(-q)\middle|O_{8g}\middle|B(p-q)\right\rangle = \frac{-i}{\pi^2 f_\pi f_B m_B^2}\int_0^{s_0^\pi}\mathrm{d}s e^{-s_1/M_1^2}\int_{m_b^2}^{s_0^B}\mathrm{d}s_2 e^{(m_B^2-s_2)/M_2^2} \times \mathrm{Im}_{s_1}\,\mathrm{Im}_{s_2}\,F\left(s_1,s_2,m_B^2\right), \quad (5)$$

where F is the part of the correlation function F_α (4) proportional to the momentum $(p-k)$:

$$F_\alpha = (p-k)_\alpha F + q_\alpha\tilde{F}_1 + k_\alpha\tilde{F}_2 + \epsilon_{\alpha\beta\lambda\rho}q^\beta p^\lambda k^\rho\tilde{F}_3, \quad (6)$$

and the thresholds s_0^π and s_0^B are fitted from other sum rules, while f_π and f_B are the pion and B-meson decay constants, respectively and m_B stands for the B-meson mass. M_1 and M_2 are the Borel parameters.

3 Evaluation and results

The main task consists in computing the correlation function defined in eq. (4) in the spacelike region of the variables s_1, s_2, and Q^2. The details of this computation are described in [2]. It can be split into three parts: the hard, soft, and quark condensate contribution. This calculation includes the twist-2 and -3 hard-gluon effects as well as soft-gluon effects of twist 3 and 4. The latter are suppressed by the quark mass but not by α_s so that they become numerically of the same order as the hard gluonic ones. Also we include the quark condensate contribution as a check on the validity of the twist expansion. The calculation was done with the help of FORM [8].

The hard-gluon contribution gives one-loop diagrams whose ultraviolet divergencies are removed after subsequent Borel transformation. The soft-gluon contribution as well as the quark condensate are calculated at the tree level. The result for the matrix element is

$$A^{(O_{8g})}\left(\bar{B}_d^0\longrightarrow\pi^+\pi^-\right) \equiv \left\langle\pi^-(p)\pi^+(-q)\middle|O_{8g}\middle|B(p-q)\right\rangle = A^{(O_{8g})}_{\text{hard}} + A^{(O_{8g})}_{\text{soft}} + A^{(O_{8g})}_{\langle\bar{q}q\rangle}, \quad (7)$$

where

$$A^{(O_{8g})}_{\text{hard}} = i\frac{\alpha_s C_F}{2\pi}m_b^2\left(\frac{1}{4\pi^2 f_\pi}\int_0^{s_0^\pi}\mathrm{d}s e^{-s/M_1^2}\right)\frac{m_b^2 f_\pi}{2m_B^2 f_B} \times\int_{u_0^B}^1\frac{\mathrm{d}u}{u}e^{m_B^2/M_2^2-m_b^2/uM_2^2}\left(\frac{\varphi_\pi(u)}{u}+\text{twist 3}\right), \quad (8)$$

$$A^{(O_{8g})}_{\text{soft}} = -im_b^2\left(\frac{1}{4\pi^2 f_\pi}\int_0^{s_0^\pi}\mathrm{d}s e^{-s/M_1^2}\right)\left(\frac{20 f_\pi\delta_\pi^2}{m_B^2 f_B}\right. \times\left.\int_{u_0^B}^1\mathrm{d}u e^{m_B^2/M_2^2-m_b^2/uM_2^2}\bar{u}\left(u+\frac{m_b^2}{m_B^2}\right)\right), \quad (9)$$

$$A^{(O_{8g})}_{\langle\bar{q}q\rangle} = i\frac{\alpha_s C_F}{3\pi}m_b^2\left(\frac{-\langle\bar{q}q\rangle}{f_\pi m_b}\right)\left(\frac{m_b^2 f_\pi}{2m_B^2 f_B}\int_{u_0^B}^1\frac{\mathrm{d}u}{u}\right. \times\left. e^{m_B^2/M_2^2-m_b^2/uM_2^2}\left(\frac{\varphi_\pi(u)}{u}+\text{twist 3}\right)\right), \quad (10)$$

where $u_0^B = m_b^2/s_0^B$ and we have employed the asymptotic distribution amplitudes for the soft part. The twist-2 distribution amplitude φ_π as well as the omitted terms of higher twist can be found in [2]. The parameter δ_π^2 determines the normalization of the twist-4 quark-antiquark-gluon distribution amplitude and $\langle\bar{q}q\rangle$ is the quark condensate density.

From the formulae above, it is seen that although the soft contribution is suppressed by the second power of the heavy-quark mass with respect to the hard-gluon contribution, a factor of 20 invalidates neglecting this formally higher-order term. In fact, numerically the hard and soft contributions are of similar size.

Since we have at our disposal the exact sum rule for the matrix element, we can expand it in the heavy-quark mass. Such an analysis is interesting since it helps see the importance of finite-mass effects. The heavy-mass limit can be systematically performed once we make explicit the scaling of the different quantities with the b-quark mass:

$$m_B = m_b + \bar{\Lambda}, \qquad s_0^B = m_b^2 + 2m_b\omega_0, \qquad M_2^2 = 2m_b\tau, \qquad f_B = \frac{\hat{f}_B}{\sqrt{m_b}}, \quad (11)$$

where $\bar{\Lambda}$, ω_0, τ, and $\hat{f}_B$ are the parameters independent of the heavy-mass scale. On substitution of these in the full result (7) and keeping only the leading terms of the each of the hard, soft, and quark condensate contributions we find that the three contributions scale as follows,

$$A^{(O_{8g})}_{\text{hard}}\sim\sqrt{m_b}, \quad A^{(O_{8g})}_{\text{soft}}\sim\frac{\delta_\pi^2}{m_b^{3/2}}, \quad A^{(O_{8g})}_{\langle\bar{q}q\rangle}\sim m_b^{-1/2}. \quad (12)$$

Rather than dealing with the matrix element $A^{(O_{8g})}$ itself, one can reduce the theoretical uncertainty by considering the ratio of this element to the factorizable amplitude for $B\to 2\pi$ decay, which comes from the O_1 operator, and is expressed as

$$A_E^{(O_1)} \equiv \left\langle\pi^-(p)\pi^+(-q)\middle|O_1\middle|B(p-q)\right\rangle_E = im_B^2 f_\pi f^+_{B\pi}(0)\,. \quad (13)$$

The form factor $f^{+}_{B\pi}$ is found in the LCSR [9–11]. We thus define

$$r_{8g} = A^{(O_{8g})}/A^{(O_1)} \tag{14}$$

and give a numerical predicition for the ratio r_{8g} due to its smaller sensitivity to input parameters compared to the matrix element $A^{(O_{8g})}$. Doing the numerical analysis we use the asymptotic forms of the distribution amplitudes. The detailed formulae for these can be found in [2]. The inclusion of non-asymptotic corrections, however, does not lead to any significant modifications. The numerical input in the expression for the matrix element $A^{(O_{8g})}$ includes the Borel parameters M_1^2, M_2^2, the duality cutoffs s_0^π, s_0^B, the decay constants f_π and f_B, the quark pole mass m_b, the parameters of the distribution amplitudes, δ_π^2, μ_π, and the quark condensate $\langle \bar{q}q \rangle$.

The pion channel parameters fitted from the two-point SVZ sum rule [3] for $f_\pi = 132\,\mathrm{MeV}$ are $s_0^\pi = 0.7\,\mathrm{GeV}^2$ and $M_1^2 = 0.7$–$1.2\,\mathrm{GeV}^2$. The b-quark pole mass is taken at $m_b = 4.7 \pm 0.1\,\mathrm{GeV}$, consistent with the recent average of the $\overline{MS}$ mass $m_b(m_b) = 4.24 \pm 0.11\,\mathrm{GeV}$ [12] and then from the QCD sum rule for f_B we have $s_0^B = 35 \pm 2\,\mathrm{GeV}^2$. The renormalization scale is allowed to vary from $\mu_b/2$ to $2\mu_b$, where $\mu_b = \sqrt{m_B^2 - m_b^2} \simeq 2.4\,\mathrm{GeV}$. This range is chosen due to the two scales present in the problem, the Borel parameters $M_1^2 = 0.7$–$1.2\,\mathrm{GeV}^2$ and $M_2^2 = 10 \pm 2\,\mathrm{GeV}^2$. For the parameters of the distribution amplitudes, we take $\delta_\pi^2(\mu_b) = 0.13\,\mathrm{GeV}^2$ and $\mu_\pi(\mu_b) = 2\,\mathrm{GeV}$, and for the quark condensate $\langle \bar{q}q \rangle(\mu_b) = (-260\,\mathrm{MeV})^3$. With these ranges of the parameters, one obtains the following prediction for the ratio r_{8g}:

$$\begin{aligned} r_{8g} = 0.02 = 0.027\ (\text{hard}) - 0.015\ (\text{soft}) \\ + 0.008\ (\text{condensate}), \end{aligned} \tag{15}$$

see [2] for the discussion of the theoretical uncertainty on this number. In eq. (15) the contributions from the different parts of the amplitude are indicated. Evidently one cannot neglect the soft-gluon contribution.

Having at hand the expression for the matrix elements and for the ratio r_{8g} valid for a finite m_b, it is easy to see how important finite-mass effects are. For the ratio r_{8g}, the limit of an infinite heavy-quark mass gives

$$r_{8g}|_{m_b \to \infty} = 0.04. \tag{16}$$

Clearly, the infinite-mass limit is not very close to the exact value of the prediction. This is another incentive to study the hadronic B decays with the sum rule method, making it complementary to the factorization approach.

4 Conclusions

We have calculated the leading-order contribution to the matrix element of the chromomagnetic dipole operator O_{8g} relevant to the $B \to \pi\pi$ decay. The method used is based on the light-cone sum rules. Both hard and soft gluons contribute to this quantity and our method has allowed us to take these two effects into account in a systematic way. It has been found that the formally suppressed soft-gluon contribution is of the same order as the hard-gluon one. We have also computed the quark condensate contribution which is in fact small compared to the leading one. We give a numerical prediction for the ratio of the operator O_{8g} matrix element and the factorizable amplitude. It is pointed out that the finite-mass effects modify the infinite-mass limit rather substantially.

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